STAT/MA 41600
In-Class Problem Set #39 part 2: November 17, 2017
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Problem Set 39 part 2 Answers

1. We recall from the previous problem set that $\mathrm{Cov}(X_i, X_i) = 12/169$, but $\mathrm{Cov}(X_i, X_i)$ is the variance of X_i. We also recall that $\mathrm{Cov}(X_i, X_j) = -4/2873$. We conclude that the correlation between X_i and X_j is $\mathrm{Cov}(X_i, X_j)/\sqrt{\mathrm{Var}(X_i)\,\mathrm{Var}(X_j)} = (-4/2873)/\sqrt{(12/169)^2} = -1/51 = -0.0196$.

2. We recall from the previous problem set that $\mathrm{Cov}(X_i, X_i) = 1/4$, but $\mathrm{Cov}(X_i, X_i)$ is the variance of X_i. We also recall that $\mathrm{Cov}(X_i, X_j) = -1/20$. We conclude that the correlation between X_i and X_j is $\mathrm{Cov}(X_i, X_j)/\sqrt{\mathrm{Var}(X_i)\,\mathrm{Var}(X_j)} = (-1/20)/\sqrt{(1/4)^2} = -1/5 = -0.2$.

3. We already learned in Problem Set 28 that $\mathbb{E}(X) = 4/3$ and $\mathbb{E}(Y) = 2/3$.

Now we compute $\mathbb{E}(X^2) = \int_0^2\int_{2y-4}^{8-4y}(x^2)(1/12)\,dx\,dy = \int_0^2 (x^3/3)(1/12)|_{x=2y-4}^{8-4y}\,dy = \int_0^2 (24)(2-y)^3(1/12)\,dy = 8$ so $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 8 - (4/3)^2 = 56/9$. We also compute $\mathbb{E}(Y^2) = \int_0^2\int_{2y-4}^{8-4y}(y^2)(1/12)\,dx\,dy = \int_0^2 (xy^2)(1/12)|_{x=2y-4}^{8-4y}\,dy = \int_0^2 (12-6y)(y^2)(1/12)\,dy = 2/3$ so $\mathrm{Var}(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2 = 2/3 - (2/3)^2 = 2/9$.

During the last problem set, we computed $\mathrm{Cov}(X, Y) = -2/9$.

So we conclude that the correlation between X and Y is $\mathrm{Cov}(X, Y)/\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)} = (-2/9)/\sqrt{(56/9)(2/9)} = -\sqrt{7}/14 = -0.1890$.

4. We already saw in the previous problem set that $\mathbb{E}(X) = 3/2$, $\mathbb{E}(Y) = 1/2$, and $\mathrm{Cov}(X, Y) = 1/20$.

Now we compute $\mathbb{E}(X^2) = \int_0^2\int_0^x (x^2)(3/4)(x-y)\,dy\,dx = \int_0^2 (3/4)(yx^3 - y^2x^2/2)|_{y=0}^{x}\,dx = \int_0^2 (3/4)(x^4/2)\,dx = (3/4)(x^5/10)|_{x=0}^{2} = 12/5$, so $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 12/5 - (3/2)^2 = 3/20$. Also we compute $\mathbb{E}(Y^2) = \int_0^2\int_0^x (y^2)(3/4)(x-y)\,dy\,dx = \int_0^2 (3/4)(xy^3/3 - y^4/4)|_{y=0}^{x}\,dx = \int_0^2 (3/4)(x^4/12)\,dx = (3/4)(x^5/60)|_{x=0}^{2} = 2/5$, so $\mathrm{Var}(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2 = 2/5 - (1/2)^2 = 3/20$.

So we conclude that the correlation between X and Y is $\mathrm{Cov}(X, Y)/\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)} = (1/20)/\sqrt{(3/20)(3/20)} = 1/3$.