STAT/MA 41600

In-Class Problem Set #42: December 1, 2017

1. Define $f(x) = x^2/72$ for $0 < x < 6$ and $f(x) = 0$ otherwise. Suppose X_1, X_2, X_3 are independent, continuous random variables that each have probability density function $f(x)$.
1a. Find the density of $X_{(1)} = \min(X_1, X_2, X_3)$.
1b. Compute $\mathbb{E}(X_{(1)})$.
1c. Find the density of the second order statistic, $X_{(2)}$, i.e., the second-smallest one.
1d. Compute $\mathbb{E}(X_{(2)})$.

2. Same setup as in **1**.
2a. Find the density of $X_{(3)} = \max(X_1, X_2, X_3)$.
2b. Compute $\mathbb{E}(X_{(3)})$.
2c. Sanity check: We can compute $\mathbb{E}(X_1) = \mathbb{E}(X_2) = \mathbb{E}(X_3) = \int_0^6 (x)(x^2/72)\,dx = 9/2$ (just trust me; or check it yourself if you want to).

We know that $X_1 + X_2 + X_3 = X_{(1)} + X_{(2)} + X_{(3)}$. Therefore, we have $\mathbb{E}(X_{(1)}) + \mathbb{E}(X_{(2)}) + \mathbb{E}(X_{(3)}) = \mathbb{E}(X_{(1)} + X_{(2)} + X_{(3)}) = \mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 9/2 + 9/2 + 9/2 = 27/2$. So please make sure your answers to **1b**, **1d**, and **2b** sum to 27/2 too.

3. Let Y_1, Y_2, Y_3 be three independent, continuous random variables, each of which are uniformly distributed on the interval $(0, 10)$. Let $Y_{(2)}$ denote the 2nd-smallest value (i.e., the middle value). Find the density of $Y_{(2)}$ and the expected value of $Y_{(2)}$.

4. Consider three independent Exponential random variables, X_1, X_2, and X_3, which each have mean 5. What are the expected value and variance of $X_{(1)}$, i.e., of the minimum of these three random variables?