STAT/MA 41600
In-Class Problem Set #43: December 4, 2017

1. Suppose that X is an Exponential random variable with λ.
1a. Find the moment generating function $M_X(t)$ of X.
1b. Compute $M'_X(0)$. Hint: You should get $1/\lambda$ for your answer, since $M'_X(0) = \mathbb{E}(X)$.
1c. Compute $M''_X(0)$. Hint: You should get $2/\lambda^2$ for your answer, since $M''_X(0) = \mathbb{E}(X^2)$.
(We learned these facts in 1b and 1c on October 27, 2017, in the notes for Problem Set 32.)

2. Same setup as in **1**.
2a. Compute $\mathbb{E}(X^3) = M'''_X(0)$. (This would previously have taken 3 integrations by parts!)
2b. Compute $\mathbb{E}(X^4) = M''''_X(0)$. (This would previously have taken 4 integrations by parts!)
2c. Can you find a general formula for $\mathbb{E}(X^n) = M_X^{(n)}(0)$?

3. Suppose that X is a Chi-squared random variable with parameter k. Then X has moment generating function $(1-2t)^{-k/2}$. (Technical point: this MGF is valid for $t < 1/2$.)
3a. Find $\mathbb{E}(X)$.
3b. Find $\mathbb{E}(X^2)$.
3c. Use your answers above to find $\text{Var}(X)$.

4. Suppose random variable X has probability mass function $P(X = x) = (125/156)(1/5)^x$, for integers $0 \le x \le 3$.

a. Verify that this is a valid probability mass function.

b. Manually compute the expected value of X.

c. Find the moment generating function $M_X(t)$ of X. (If you think for a moment, it is possible to write $M_X(t)$ without using any summation signs or addition symbols.)

d. Compute $M'_X(0)$. Hint: Your answer should agree with your answer for **4b**.