

STAT/MA 41600
In-Class Problem Set #19: September 28, 2018

1. Consider 5 Wookies who are running toward 20 tie-fighter ships, hoping to use lightspeed to escape from a planet that is about to explode. They want to board the 20 ships and escape! Little do they know that the Emperor has already foreseen their plans, and disabled the engine on 7 of the ships, so that only 13 of the ships are operational.

[Unfortunately for the Wookies, only 1 Wookie can fit into each tie-fighter, and they only have time to try 1 tie-fighter each, before the planet explodes...so the Wookies are trying the tie-fighters without replacement. Assume that the Wookies are really in a hurry (obviously!) so all of the possible outcomes are equally likely. The Emperor is crafty, and he made it hard to tell which tie-fighters are operational.]

1a. What is the probability that only 1 Wookie escapes?

1b. What is the probability that at least 3 Wookies escape?

2. A certain gold-colored droid famously claims to be “fluent in over six million forms of communication.” (We will assume that the droid knows *exactly* six million languages.)

The droid is trying to use his languages to communicate with the trash compactor, to save his three friends from getting crushed.

Among the six million languages, the trash compactor only recognizes 1 million out of the 6 million languages.

The droid only has time to try 10 of his 6 million languages, before the trash compactor seals the fate of his three friends!

2a. Write an exact expression for the probability that 0 of his 10 attempted languages will work (his friends will unfortunately be crushed in this case).

2b. Write an exact expression for the probability that exactly 3 of his 10 attempted languages will work.

2c. Make a Binomial approximation to the probabilities in **2a** and **2b**, and compute the value of these approximations.

3. Ewoks enjoy having bonfires to celebrate, and they like to dance too. This is a dangerous combination, because sometimes they get too close to the fire and they burn their fur.

Consider 15 Ewoks who are dancing very energetically near a fire. Suppose that exactly 6 of the Ewoks have actually burned themselves (the other 9 are not burned). The doctors are trying to help them, but it is dark so they cannot see well. They randomly treat 10 of the 15 Ewoks.

Let X denote the number of the 10 treated Ewoks who were actually burned.

3a. What are the mean and the variance of X ?

3b. Find the probability that X is an odd number.

4. Consider a Jedi training session in which 30 young Jedis participate. Exactly 10 of the Jedis are prepared to lift heavy rocks with the Force, but the other 20 are unprepared.

On the examination day, the Jedi masters will examine half of the young Jedis (i.e., 15 out of 30 of them).

4a. Find the probability that they find exactly 8 of the 10 prepared young Jedis, among the 15 young Jedis in their test.

4b. Find the probability that they find at least 8 of the 10 prepared young Jedis, among the 15 young Jedis in their test.